# TITLE: A MARKOV CHAIN MODEL TO DETERMINE THE BEHAVIOUR OF DIFFERENT COMPANY SHARE PRICES

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# A MARKOV CHAIN MODEL TO DETERMINE THE BEHAVIOUR OF DIFFERENT COMPANY SHARE PRICES

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**Abstract**

Market value of the share is the yard stick to measure the performance of that company, to which the share belongs to. Further share price is the most important concern in market research. Hence to analyse these share prices of different companies, we have collected time series data representing the prices of each share from January 2012 to June 2013. In the present paper we have to consider four company shares and proposed a Markov chain model to these companies individually and obtained steady-state solutions, after forming transition Probability matrices (t.p.m.s). For the formation Markov chain model, the following three states considered namely;

State "-1" represents a decrease in the face value of the share from $i^{th}$ *day to* $(i+1)^{th}$ *day*

State "0" represents no change in the face value of the share from $i^{th}$ *day to* $(i+1)^{th}$ *day* .

State "1" represents an increase in the face value of the share from $i^{th}$ *day to* $(i+1)^{th}$ *day* .

Let the closing value of the share at $i^{th}$ day is represented by $X_i$ is a stochastic variable which forms time series where $'i'$ represents the working day of the share market. Conclusions and comparisons are made among different companies based on the steady-state probabilities obtained. Ergodic property of the steady-state is also discussed. Next to study the behavior of market value of a share through simulation technique.

**Keywords:** Markov chain model, Transition probability matrix (t.p.m.), Steady- state solutions, Ergodicity, Monte-Carlo simulation technique.

## Introduction

Investments in share market have attractive returns, but at the same time it involves hidden risk factors. Hence one has to think properly before investing (or) purchasing a company share. A well experienced investor can predict the prosperity of a share based on the company's past history, fiscal policies of the Government, investment patterns in the past and in the future. This type of prediction is not a scientific one and often may go wrong because the decision taken is based on the past experience of the investor, but not done using scientific methods of predictions. Scientific predictions are also based on collection of the past data and analysis of the same through sophisticated techniques. Here also, there is a possibility of the decisions failing but the chances are very remote say 5% (or) 1% (or) still less, based on the techniques used. In this connection it is worth to mention that use of more recent Statistical techniques is very useful and helpful to take appropriate decisions by a common investor. Thus there is a necessity to apply Statistical techniques to analyze the share market data collected through proper resources. This leads to a new research field known as "Market Research".

Market Research, now a day, occupies a vital place in modern research because it provides solutions to business problems through an appropriate model. A model is a replica of the real situation. If a business problem is represented through a mathematical equation then it is called 'Business Mathematical Model (B.M.M)'. Thus Business Mathematical Models play an important role in Market Research and are mainly based on the assumptions considered by the researcher. Such assumptions are necessary and inevitable to apply well known Mathematical tools and techniques, through which one can get solutions to the model proposed. But in reality these assumptions sometimes may not be true. For instance, generally one can assume a linear relationship between demand and price. But this linearity may not hold good for all the items in the market. For example, L.P.Gas cylinder's price may not increase depending upon the demand. If demand is more, supply of the cylinders may perhaps be delayed, but cost may not increase. Thus the basic assumption of linearity is invalid here. Similarly, in some other situations a non-linear relationship may hold well between the variables under consideration. Hence, the proposed model is to be modified by incorporating such new assumptions into the model, so that the modified model is closer to the reality. In Market Research probabilistic models proposed by Researchers are to be updated, because the variables under consideration inherently have stochastic nature**.** Thus stochastic models are more appropriate to deal with problems arising in Market Research than probabilistic models. This is very essential particularly to determine the behavior of the Market Value of a Share.

Market value of a share is very sensitive and depends on many factors, their interrelationships, and fiscal policies taken by the Government/ Reserve Bank of India (RBI)/Stock Exchange Board of India (SEBI) and so on. Thus stochastic modeling is very essential in Market Research to detect the behavior of the market value of a share. In stochastic modeling, we assume that the parameters of the distribution under study are functions of time variable't', whereas; these parameters are assumed as constants in 'Probabilistic models'.

In the present paper a Markov Chain model[1] is applied to predict the behaviour of each Company share by defining the following three states namely,

State "-1" represents a decrease in the Market value of the share from $i^{th}$ to $(i+1)^{th}$ day.

State "0" represents no change in the Market value of the share from $i^{th}$ to $(i+1)^{th}$ day.

State "1" represents an increase in the Market value of the share from $i^{th}$ to $(i+1)^{th}$ day.

The transition probability matrices (t.p.m.s.)[2] are formed, steady-state solutions[3] are obtained and ergodicity property[4] of the above '3' states are discussed. Next to study the behavior of market value of a share through Monte-Carlo simulation technique and to compare these simulated results with the reality then to apply $\chi^2$ –test for testing validity.

## A Markov Chain Model for the determination of the behaviour of Company Shares.

A Stochastic system [11]is called a Markov Process if the occurrence of a future state depends on the immediately preceding state and only on it. In other words future depends only on present and present depends only on the immediate past. This property is popularly known as *"Memory Less"* property [5][8][9].

Here S= a countable set, T = $\{0,1,2,3,....\}$. The Stochastic Process $\{X_n, n = 0,1,2,3,.....\}$ is called a Markov Chain, if, for $j, k, j_1, ......j_{n-1} \in N$ (or any subset of I),

$P\{X_n = k / X_{n-1} = j, X_{n-2} = j_1, ......X_0 = j_{n-1}\} = P\{X_n = k / X_{n-1} = j\} = p_{jk}$ (say) whenever the first member is defined. The outcomes are called the states of the Markov Chain.

Let $'X_i'$ represents the closing value of the share on $'i^{th}'$working day of the share market. As $'X_i'$ changes day to day and is influenced by many independent variables,$'X_i'$ has stochastic in nature inherently present and to predict this nature a Markov Chain model is considered, where the behaviour of the share on $(i+1)^{th}$ day depends on $'i^{th}'$ day price. Further, it is assumed that past behaviour as no effect in determining the future behaviour of a share. Thus the following Markov Chain is considered with state space S = $\{-1,0,1\}$ and Index set 'T' is considered as a discrete variable taking the values $i$= 1, 2, 3… where $'i'$ represents the working day of the share market. Thus we have the following t.p.m.s. of size $(3\times3)$ defined as

$$P = \text{From} \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array} \overset{\text{To}}{\begin{array}{ccc} -1 & 0 & 1 \\ \end{array}} \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

Where $p_{ij} = P\left\{X_{i+1} = j \middle/ X_i = i\right\}, \; i \neq j, \; (i, j) \in I$

Using *Chapman-Kolmogorov equation* [6][7][12] higher order transitions are calculated as $P^n$ = $P \times P \times ...... \times P \;\; (n - times)$ = $(P)^n$

Limiting probabilities are steady-state probabilities denoted by 'V' is obtained as

V = $\lim_{n\to\infty} p_{ij}^{(n)}$ = $\lim_{n\to\infty} (P)^n$

### Calculation of Higher Transition Probability Matrices

Based on the data collected the following '9' transition days calculated and given in the following table (1):

**Table (1) - Frequencies of different transitions under study**

| S.No. | Transitions | Explanation |
|---|---|---|
| 1 | −1 to −1 | Two consecutive decreasing days in the Share Prices. |
| 2 | 1 to 0 | Previous day a decreased and the next day maintain the same price. |
| 3 | 1 to 1 | A decrease in the previous day and an increase in the next day. |
| 4 | 0 to 1 | No change in the previous day and a decrease in the next day |
| 5 | 0 to 0 | No change in both days. |
| 6 | 0 to 1 | No change in the previous day and an increase in the next day. |
| 7 | 1 to −1 | An increase in the previous day and a decrease in the next day. |
| 8 | 1 to 0 | An increase in the previous day and no change in the next day. |
| 9 | 1 to 1 | Two consecutive increasing days in the Share Prices. |

Using the above table (1) the transition frequency matrix for each Company is calculated and is given below:

Transition frequency matrix for Bajaj Auto Limited.

$$M_{Bajaj} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 88 & 6 & 88 \\ 3 & 0 & 6 \\ 90 & 3 & 91 \end{bmatrix} \end{array}$$

Transition frequency matrix for Reddy's Lab.

$$M_{\mathrm{Re}ddy} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 59 & 20 & 80 \\ 21 & 11 & 16 \\ 78 & 17 & 73 \end{bmatrix} \end{array}$$

Transition frequency matrix for Hero Motor Corporation Limited.

$$M_{HROM} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 93 & 17 & 70 \\ 18 & 3 & 12 \\ 68 & 13 & 81 \end{bmatrix} \end{array}$$

Transition frequency matrix for Infosys Limited.

$$M_{Infosys} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 84 & 15 & 71 \\ 17 & 2 & 12 \\ 69 & 14 & 91 \end{bmatrix} \end{array}$$

Using the above frequency matrices, one-step transition probability matrices are formed using the formula explained as follows:

$$P_{ij} = \frac{a_{ij}}{\sum_{j=1}^{3} a_{ij}}, (i, j) = 1, 2, 3$$

Using the above equation, we get one-step transition probability matrices (t.p.m.s) are formed for each Company and are given as follows:
Transition probability matrix for Bajaj Auto Limited.

$$P_{Bajaj} = \begin{matrix} & \begin{matrix} -1 & 0 & 1 \end{matrix} \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} 0.4835 & 0.0330 & 0.4835 \\ 0.3333 & 0.0000 & 0.6667 \\ 0.4891 & 0.0163 & 0.4946 \end{bmatrix} \end{matrix}$$

Transition frequency matrix for Reddy's Lab.

$$P_{\mathrm{Re}ddy} = \begin{matrix} & \begin{matrix} -1 & 0 & 1 \end{matrix} \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} 3711 & 0.1258 & 0.5031 \\ 0.4375 & 0.2292 & 0.3333 \\ 0.4643 & 0.1012 & 0.4345 \end{bmatrix} \end{matrix}$$

Transition probability matrix for Hero Motor Corporation Limited.

$$P_{HROM} = \begin{matrix} & \begin{matrix} -1 & 0 & 1 \end{matrix} \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} 0.5167 & 0.0944 & 0.3889 \\ 0.5455 & 0.0909 & 0.3636 \\ 0.4198 & 0.0802 & 0.5000 \end{bmatrix} \end{matrix}$$

Transition probability matrix for Infosys Limited.

$$P_{Infosys} = \begin{matrix} & \begin{matrix} -1 & 0 & 1 \end{matrix} \\ \begin{matrix} -1 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} 0.4941 & 0.0882 & 0.4176 \\ 0.5484 & 0.0645 & 0.3871 \\ 0.3965 & 0.0805 & 0.5230 \end{bmatrix} \end{matrix}$$

One can observe that the above formulated transition probability matrices are Stochastic matrices. Critically comparing the one-step transition probability matrices of different Companies we can conclude that

1. Bajaj Auto Ltd., has the highest probability i.e., 0.6667 for the transition from 0 to 1 which indicates that Bajaj Auto Ltd., Company Share has more probability having increasing tendency from previous day to present day.
2. In Reddy's Lab. has transition from 1 to –1 is highest i.e., 0.4643 which means that probability for a decreasing tendency is more in Reddy's Lab. than other transition probabilities.

3. In case of Hero Motor Corporation Ltd. also which has highest probability for the transition state 0 to –1 i.e., 0.5455, which means that probability for a decreasing tendency is more in Hero Motor Corporation Ltd. than other transition probabilities.
4. When we compare the probabilities for Infosys Ltd. Company share it is highest for transition from 0 to -1 i.e., 0.5484 whose tendency is similar to Hero Motor Corporation Ltd.

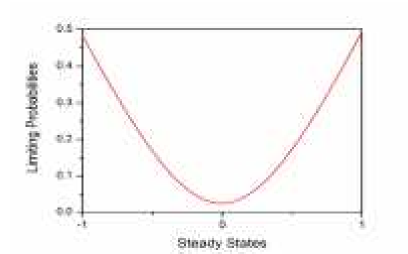
Finally we conclude that Bajaj Auto Ltd., showing upward tendency (Bulls tendency), where as Reddy's Lab., Hero Motor Corporation Ltd. & Infosys Ltd. are showing downward tendency (Bears tendency).

**Limiting behaviour of the Shares**

In order to study the limiting behaviour of the above shares, *steady-state solutions* are obtained and by using Chapman-Kolmogorov equations, we obtained the following matrices as the limit $n \to \infty$.

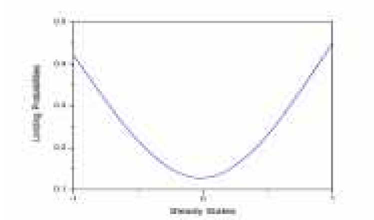
**For Bajaj Auto Limited**:

$$P_{Bajaj}^{4} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 0.4825 & 0.0239 & 0.4932 \\ 0.4825 & 0.0239 & 0.4932 \\ 0.4825 & 0.0239 & 0.4932 \end{bmatrix} \end{array}$$



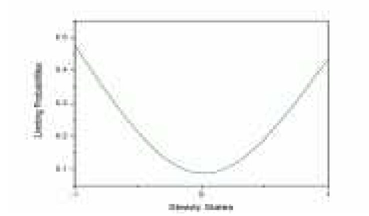

**For Reddy's Lab:**

$$P_{\mathrm{Re}ddy}^{6} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 0.4214 & 0.1278 & 0.4503 \\ 0.4214 & 0.1278 & 0.4503 \\ 0.4214 & 0.1278 & 0.4503 \end{bmatrix} \end{array}$$



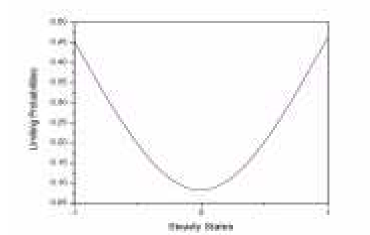
**For Hero Motor Corporation Ltd. (HROM):**

$$P_{HROM}^{6} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 0.4769 & 0.0878 & 0.4348 \\ 0.4769 & 0.0878 & 0.4348 \\ 0.4769 & 0.0878 & 0.4348 \end{bmatrix} \end{array}$$



**For Infosys Tech:**

$$P_{Infosys}^{6} = \begin{array}{c} \\ -1 \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ \begin{bmatrix} 0.4530 & 0.0825 & 0.4636 \\ 0.4530 & 0.0825 & 0.4636 \\ 0.4530 & 0.0825 & 0.4636 \end{bmatrix} \end{array}$$



**Determination of the behavior of Market value of Share through Simulation Technique.**

Markov Chain model is applied to simulate the behavior of each Company share by the '9' transitions based on the three states namely $'-1', '0', '1'$. Now the behavior of these shares is simulated by using the following Monte-Carlo simulation technique [13].

**Monte-Carlo Simulation Technique:**

- Count the frequency for different transitions to be simulated
- Determine the probability distribution for each transition in step 1 and establish the cumulative distribution function.
- Set up the table and assign tag numbers, with the help of cumulative distribution function.
- Generate random numbers and choose the corresponding tag number. Then select the variable value corresponding to the tag number.
- Generate random numbers at many numbers of trials (given in the problem) and compute the values for different trials. The optimal solution will be the average values of different trials.
- Now these simulated results are compared with actual daily Market values from 1st July 2013 to 13th July 2013. Next to apply $\chi^2$ –test for testing validity.

**Comparison of the behavior of different Companies through Simulated Results.**

In order to simulate the behavior of the above Company Shares, probabilities, cumulative probabilities and ranges of Random Numbers for four Companies are given by the following tables:

**Table 2: Empirical Probability Distribution and Ranges for Random Numbers for Bajaj. Company.**

| State from to | Frequency | Probability | cumulative probability | Ranges of Random numbers |
|---|---|---|---|---|
| 1' to ' 1' | 88 | 0.2347 | 0.2347 | 0000 2346 |
| -1 to 0 | 6 | 0.0160 | 0.2507 | 2347-2506 |
| 1' to 1 | 88 | 0.2347 | 0.4854 | 2507 4853 |
| 0 to '-1' | 4 | 0.0080 | 0.4944 | 4854-4943 |
| 0 to 0 | 0 | 0 | 0.4944 | 4944 |
| 0 to 1 | 6 | 0.0160 | 0.5094 | 4945-5094 |
| 1 to ' 1' | 90 | 0.2100 | 0.7479 | 5095 7179 |
| 1 to 0 | 4 | 0.0080 | 0.7574 | 7480-7559 |
| 1 to 1 | 91 | 0.2126 | 1 | 7550 9999 |
| Total | 375 | 1 | | |

**Table3: Empirical Probability Distribution and Ranges for Random Numbers for Reddy's Lab.**

| State from—to | Frequency | Probability | cumulative probability | Range of Random numbers |
|---|---|---|---|---|
| 1' to ' 1' | 59 | 0.1573 | 0.1573 | 0000 1572 |
| -1 to 0 | 20 | 0.0533 | 0.2106 | 1573-2105 |
| -1' to 1 | 80 | 0.2133 | 0.4239 | 2106-4238 |
| 0 to -1' | 21 | 0.056 | 0.4799 | 4239-4798 |
| 0 to 0 | 11 | 0.0293 | 0.5092 | 4799-5091 |
| 0 to 1 | 16 | 0.0427 | 0.5519 | 5092-5518 |
| 1 to ' 1' | 78 | 0.2080 | 0.7599 | 5519 7598 |
| 1to 0 | 17 | 0.0454 | 0.8053 | 7599-8052 |
| 1 to 1 | 73 | 0.1947 | 1 | 8053 9999 |
| Total | 375 | 1 | | |

**Table4: Empirical Probability Distribution and Ranges for Random Numbers for HROM Company.**

| State from—to | Frequency | Probability | cumulative probability | Range of Random numbers |
|---|---|---|---|---|
| 1' to ' 1' | 93 | 0.2480 | 0.2480 | 0000-2479 |
| -1 to 0 | 17 | 0.0453 | 0.2933 | 2480-2932 |
| -1' to 1 | 70 | 0.1867 | 0.4800 | 2933-4799 |
| 0 to ' 1' | 18 | 0.0480 | 0.5280 | 4800-5279 |
| 0 to 0 | 3 | 0.0080 | 0.5360 | 5280-5359 |
| 0 to 1 | 12 | 0.0320 | 0.5680 | 5360-5679 |
| 1 to '-1' | 68 | 0.1813 | 0.7493 | 5680-7492 |
| 1to 0 | 13 | 0.0347 | 0.7840 | 7493-7839 |
| 1 to 1 | 81 | 0.2160 | 1 | 7840-9999 |
| Total | 375 | 1 | | |

**Table5: Empirical Probability Distribution and Ranges for Random Numbers for Infosys Ltd.**

| State from---to | Frequency | Probability | cumulative probability | Range of Random numbers |
|---|---|---|---|---|
| -1' to '-1' | 84 | 0.2240 | 0.2240 | 0000-2239 |
| -1 to 0 | 15 | 0.0400 | 0.2640 | 2240-2639 |
| '1' to 1 | 71 | 0.1893 | 0.4533 | 2640-4532 |
| 0 to '-1' | 17 | 0.0453 | 0.4986 | 4533-4985 |
| 0 to 0 | 2 | 0.0053 | 0.5039 | 4986-5038 |
| 0 to 1 | 12 | 0.0320 | 0.5359 | 5039-5358 |
| 1 to'-1' | 69 | 0.1840 | 0.7199 | 5359-7198 |
| 1to 0 | 14 | 0.0374 | 0.7573 | 7199-7572 |
| 1 to 1 | 91 | 0.2427 | 1 | 7573-9999 |
| Total | 375 | 1 | | |

For the purpose of comparison of the proposed models with the reality Random Numbers are generated using procedures available in the latest version of MS-Excel. For this purpose, we have generated '30' Random numbers in each share because we have 30 working days for Stock Market from 1st July 2013 to 13th August 2013.

### Simulation results with actual results

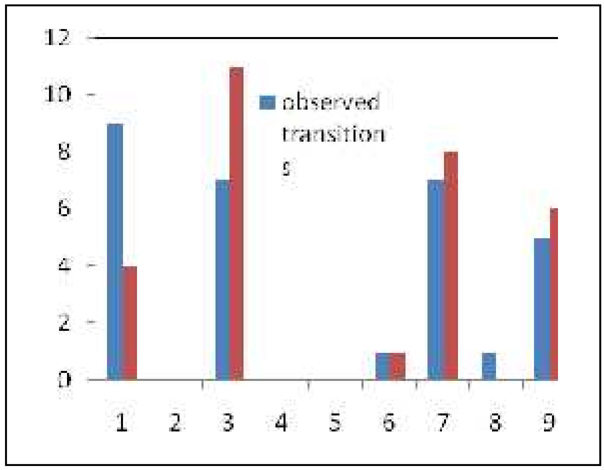
Using the Random numbers for frequencies of '9' transitions from states to states are calculated and calculated frequencies for each Company for set of Random numbers is given and to verify the closeness of Simulated frequencies with actual frequencies in each Company Share $\chi^2$-test statistic along with critical values are given in last row of respective Company table.

### Simulated Results and Test Statistics $\chi^2$ – Values for Baja Auto Limited.

***Null Hypothesis ($H_0$):*** The Markov Chain model is the best fit for company share Bajaj Auto Ltd.

Random numbers: 6578, 7337, 4572, 3437, 6916, 326, 6466, 8445, 7363, 8851, 2791, 4091, 3424, 3801, 9723, 7652, 3537, 2577, 1627, 9626, 4807, 3427, 7452, 7963, 7466, 4511, 1878, 5498, 5414, 1523.

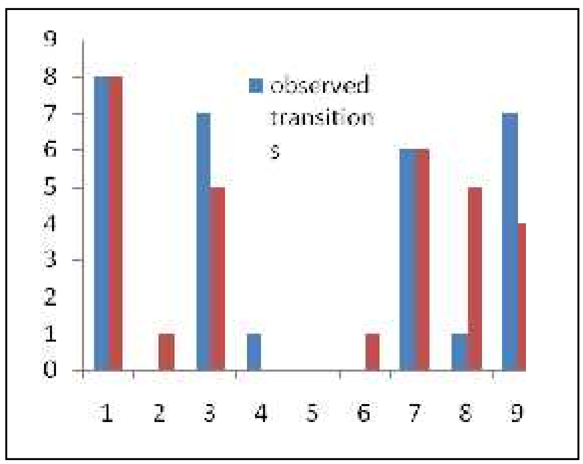
| Observed frequencies | Simulated frequencies |
|---|---|
| 9 | 4 |
| 0 | 0 |
| 7 | 11 |
| 0 | 0 |
| 0 | 0 |
| 1 | 1 |
| 7 | 8 |
| 1 | 0 |
| 5 | 6 |
| $\chi^2$ −value | 0.1778 |
| Table value | $\chi^2_{0.05}$ for 3d.f. =7.815 |




**Simulated Results and Test Statistics $\chi^2$ – Values for Reddy's Lab**

***Null Hypothesis $H_0$:*** The Markov Chain model is the best fit for the Company share Reddy's Lab.
Random numbers: 7622, 1169, 9811, 5536, 1010, 8240, 9013, 3552, 8757, 3523, 663, 7878, 1262, 2572, 2486, 7663, 7748, 7963, 805, 7587, 7001, 5446, 6588, 1603, 7086, 1591, 902, 2977.

| Observed frequencies | Simulated frequencies |
|---|---|
| 8 | 8 |
| 0 | 1 |
| 7 | 5 |
| 1 | 0 |
| 0 | 0 |
| 0 | 1 |
| 6 | 6 |
| 1 | 5 |
| 7 | 4 |
| $\chi^2$ −value | 2.31 |
| Table value | $\chi^2_{0.05}$ for 3d.f. =7.815 |

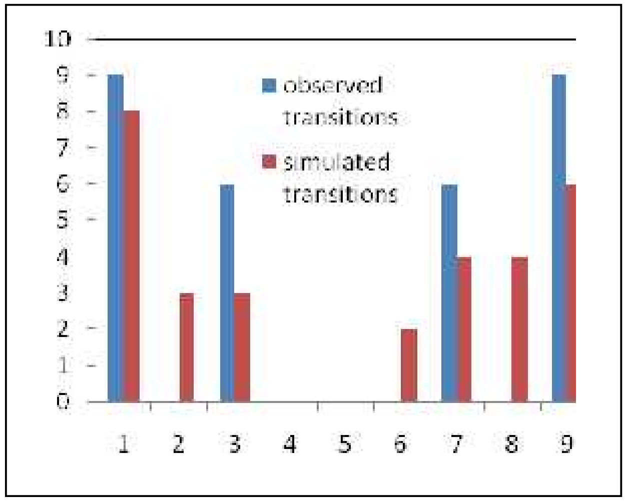



**Simulated Results and Test Statistics $\chi^2$ – Values for the Company share HROM**

***Null Hypothesis $H_0$:*** The Markov Chain model is the best fit for the Company share HROM.
Random numbers: 7622, 1169, 9811, 5536, 1010, 8240, 9013, 3552, 8757, 3523, 663, 7878, 1262, 2572, 2486, 7663, 7748, 7963, 805, 7587, 7001, 5446, 6588, 1603, 7086, 1591, 902, 2977.

| Observed | Simulated |
|---|---|

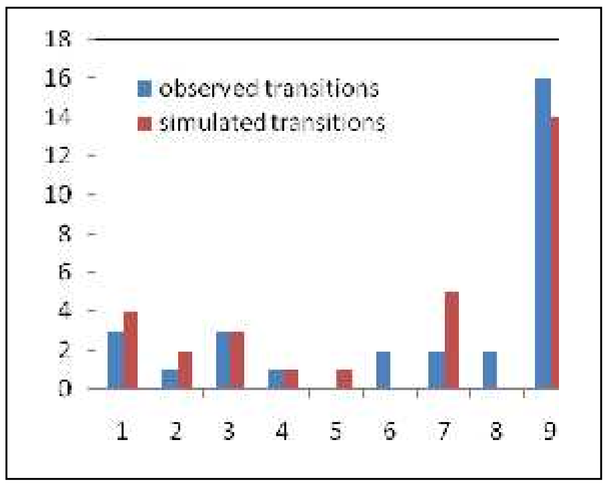
| frequencies | frequencies |
|---|---|
| 9 | 8 |
| 0 | 3 |
| 6 | 3 |
| 0 | 0 |
| 0 | 0 |
| 0 | 2 |
| 6 | 4 |
| 0 | 4 |
| 9 | 6 |
| $\chi^2$ − value | 0.225 |
| Table value | $\chi^2_{0.05}$ for 3d.f. =7.815 |




***Simulated Results and Test Statistics $\chi^2$ – Values for Infosys Company***

***Null Hypothesis $H_0$:*** The Markov Chain model is the best fit for the Company share Infosys Ltd.
Random numbers: 5894, 2538, 7654, 8195, 316, 7779, 394, 9583, 2135, 9403, 8824, 7978, 6823, 6005, 8807, 8455, 4813, 2538, 8896, 9913, 3693, 2670, 3080, 5795, 1052, 9561, 5011, 9219, 9353, 5836.

| Observed frequencies | Simulated frequencies |
|---|---|
| 3 | 4 |
| 1 | 2 |
| 3 | 3 |
| 1 | 1 |
| 0 | 1 |
| 2 | 0 |
| 2 | 5 |
| 2 | 0 |
| 16 | 14 |
| $\chi^2$ − value | 1.3523 |
| Table value | $\chi^2_{0.05}$ for 7d.f. =14.07 |




## Results and Discussion

By comparing the above matrices one can observed that the states considered here are all ergodic states because ergodicity property is achieved when at $n = 6$ in all the four Companies. Further the limiting behaviour of the Companies or the behaviour of the Companies in the run is explained through these limiting probabilities.

**Table 6: Steady-state Solutions**

| Name of the Company | Limiting probability | | |
|---|---|---|---|
| | -1 | 0 | 1 |
| Bajaj Auto Ltd. | 0.4825 | 0.0239 | 0.4932 |
| Reddy's Lab | 0.4214 | 0.1278 | 0.4503 |
| HROM | 0.4769 | 0.0878 | 0.4348 |
| Infosys Ltd. | 0.4530 | 0.0825 | 0.4636 |

It is interesting to note that the steady-state probabilities satisfy the conditions for ergodicity and forms a perfect probability distribution for each Company Share under consideration. Thus the states considered here namely, $-1, 0, 1$ are ergodic states because the limiting probabilities satisfy the ergodic conditions.

Critically comparing these probabilities one can draw the following conclusions:

- Bajaj Company share has highest limiting probability (0.4932), Reddy's Lab has highest limiting probability (0.4503) and Infosys Ltd., has highest limiting probability (0.4636) for an increasing tendency in the share prices.
- Whereas the behaviour of the share Hero Motor Corporation Ltd., has the highest limiting probability (0.4769) for the state '-1' i.e., in decreasing tendency.
- In all Companies probabilities are least for state '0', which represents no change in the share prices, its chance is very low.
- Finally observing the simulated results of different Companies with Actual results, i.e., a closer relationship between simulated results and Actual results in all cases of the above four Company shares. Thus we finally conclude that the proposed Markov Chain models based on the past data are more appropriate ones for comparison of the future behavior of these Companies under consideration.

**References**

1. BASU, A.K. (2003). "Introduction to Stochastic Processes". *Narosa Publishing House*, New Delhi.
2. MEDHI, J. (1984). "Stochastic Processes", Second Edition. *New Age International (P) Ltd., Publishers*, New Delhi.
3. SAMUEL KARLIN and HOWARD M.TAYLOR (1975) "A First Course in Stochastic Process", Second Edition, *Academic Press, Inc (London) Ltd*.
4. J.Medhi. A Markov chain model for the occurrence of dry and wet days, *Ind. J. Met. Hydro. & Geophys*.27, 431-435 (1976).
5. B.L.S.Prakasa Rao. *Asymptotic Theory of Statistical Inference*, Wiley, New York (1987).
6. S.M.Ross. *Stochastic Processes*, Wiley, New York (1983).

7. A.G.Pakes. Some conditions of ergodicity and recurrence of Markov chains, *Opns.Res*.17, 1058-1081 (1969)
8. T.W. Anderson and L.A.Goodman. Statistical inference about Markov chains, *Ann.Math. Stat*.28, 89-110 (1957)
9. A.T. Bharucha-Reid. *Elements of theory of Markov Processes and their Applications*, McGraw-Hill, New York, (1960).
10. K.L.Chung, *Markov chains with Stationary Transition Probabilities*, 2nd ed., Springer-Verlag, New York (1967).
11. W.Feller. *An Introduction to Probability Theory and its Applications*, Vol.I, 3rd ed. Wiley, New York (1968).
12. C Derman. On sequential decisions and Markov chains, *Mgmt.Sci*.29, 16-24 (1982).
13. Hamdy A. Thaha A text book of Operations Research, 9th edition, 2010.