# TORSION

→ Derivation of torsion equation and its assumptions ✓

→ Application of the equation of the hollow and solid circular saft ✓

→ Torsional rigidity ✓

→ Combined torsion and bending of circular shafts

→ Principal stress and maximum shear stresses under combined loading of bending and torsion.

# Compound Stresses and Strains

→ Two-D system

→ Stress at a point on a plane

→ Principal stresses and principal planes

→ Mohr's circle of stress and their applications

→ Two-D stress-strain system

→ Principal strains and principal axis of strain.

# Some basic terms in torsion

## (1) Shafts:

Shaft is generally a cylindrical section. It may be a solid and hollow member. This can be made up with mild steel, alloy steel and copper alloy.

## (2) Loads on shafts

(i) Torsional load
(ii) Bending load
(iii) Axial load
(iv) Combination of torsional load, Bending load and axial load.

## Torsion of shaft

→ The force is applied tengentially over a shaft.
→ The torqe (or) twisting moment may be calculated by multiplying the tengential force with the radius of shaft.

→ If shaft is subjected to two opposite turning moment, then it is called as pure tarsion. This exhibit the tendency of shearing the cross-section which is ⊥lar to longitudinal axis.

## Basic assumptions in the derivation of torsion equation:

(1) The material of the shaft is homogeneous, isotropic and perfectly elastic.

(2) The material obeys Hook's law and the stress remains within limit of proportionality.

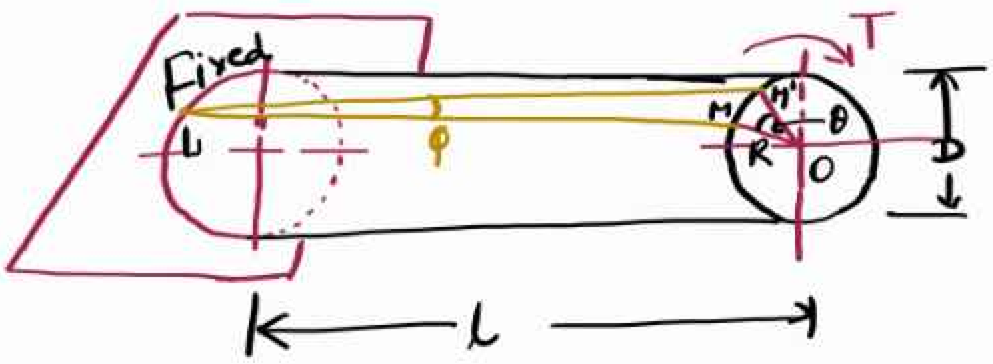
(3) The twisting moment act in the transverse plane only.

(4) The section of circular shaft remain circular after tengential loading.

(5) Parallel planes normal to the axis do not warp (or) distort after torsion.

(6) A cross-section at any axial length rotates as a rigid plane i.e. all diameters of a cross-section rotate through the same angle.

## Derivation of torsion equation



Here, $T$ is maximum twisting torque

$D$ is diameter of the shaft

$I_p$ is polar moment of Inertia

$\tau$ is shear stress

$G$ is modulus of rigidity

$\theta$ is angle of twist (radian)

$l$ is length of the shaft

From the figure,

In the given figure, shaft is fixed at one end and torque is applied at the other end. If a line LM is drawn on the shaft, it will displace to LM' due to applied torsion. The cross-section of shaft will be twisted through $\theta$ and surface is altered by angle $\varphi$.

Here, shear strain, $\varphi = \frac{MM'}{l}$ ——— ①

also, $\varphi = \frac{\tau}{G}$ ——— ②

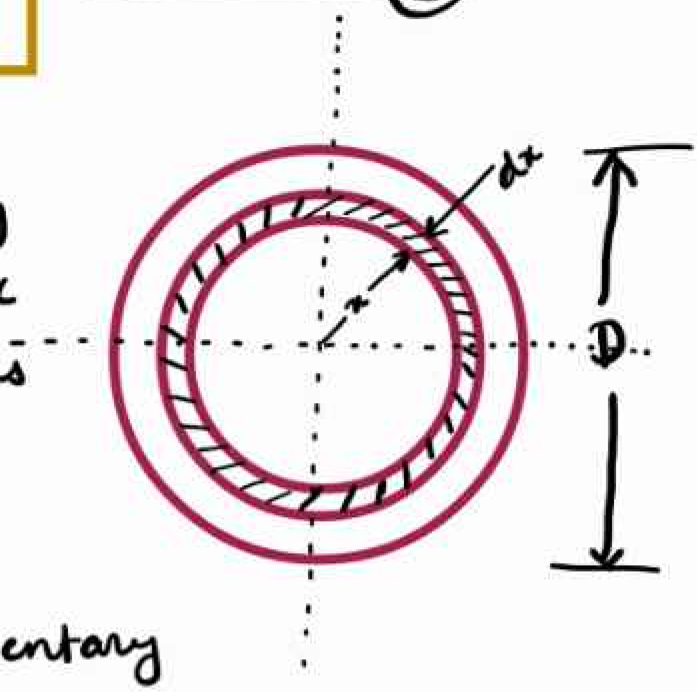
$\therefore$ Equate eq$^n$ ① & ②

$$\frac{MM'}{l} = \frac{\tau}{G}$$

$\{MM' = R\theta\}$

$$\therefore \quad \frac{R\theta}{L} = \frac{\tau}{G}$$

$$\boxed{\frac{\tau}{R} = \frac{G\theta}{L}} \quad \text{—— (3)}$$




Consider an elementary ring of thickness $dx$ at a radius $x$ and let the shear stress at this radius be $\tau_x$.

The turning force on the elementary ring $= \tau_x \cdot 2\pi x \, dx$

The turning moment due to this turning force

$$dT = \tau_x \cdot 2\pi x \cdot dx \cdot x \quad \text{—— (4)}$$

To determine total torque across cross-section.

This can be done by integrating eq^n (3) at both sides.

$$\int dT = \int_0^R \tau_x \cdot 2\pi x \cdot dx \times x$$

$$\int dT = 2\pi \int_0^R \tau_x \cdot x^2 dx \qquad \left\{ \frac{\tau}{R} = \frac{\tau_x}{x} \quad \tau_x = \frac{\tau}{R}\cdot x \right\}$$

$$= 2\pi \int_0^R \frac{\tau}{R}\cdot x \cdot x^2 dx$$

$$= 2\pi \times \frac{\tau}{R}\left[\frac{x^4}{4}\right]_0^R$$

$$= 2\pi \times \frac{\tau}{R}\cdot \frac{R^4}{4}$$

$$T = \frac{\pi \tau R^3}{2} \qquad \left\{ R = \frac{D}{2} \right\}$$

$$\boxed{T = \frac{\pi \tau D^3}{16}}$$

$$T = \frac{\tau}{R}\cdot \frac{\pi R^4}{2}$$

$$T = \frac{\tau}{R}\cdot I_p \Rightarrow \boxed{\frac{T}{I_p} = \frac{\tau}{R}} \quad \text{--- (5)}$$

from equation (3) & (5)

$$\boxed{\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}} \qquad K = \frac{T}{\theta} = \frac{GI_p}{l}$$

= torsional stiffness

$$\frac{T}{I_p} = \frac{G\theta}{L} \Rightarrow GI_p = \text{Torsional rigidity}$$

$\frac{I_p}{R} \rightarrow$ Torsional (or) polar sectional moment.

# Comparision with bending equations

$$\frac{T}{I_p} \rightarrow \frac{\text{Torsional moment}}{\text{Polar moment of Inertia}} \Rightarrow \frac{M}{I} \rightarrow \frac{\text{Bending moment}}{\text{Moment of Inertia}}$$

$$\frac{\tau}{R} \rightarrow \frac{\sigma}{y}$$

$$\frac{G\theta}{L} \rightarrow \frac{E}{R}$$

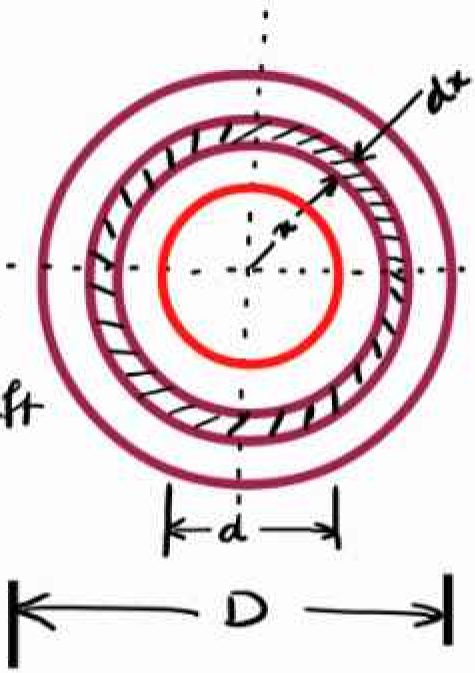
## Torque transmitted by a hollow circular shaft

Consider a hollow circular shaft subjected to a torque T.

$R \rightarrow$ outer radius of the shaft

$r \rightarrow$ inner radius of the shaft

$\tau \rightarrow$ Shear stress at radius R.




$dT$ = Turning moment on the elementary ring

$$dT = \tau_x \, 2\pi x \, dx \cdot x$$

integrating both sides

$$\int dT = \int_r^R \tau_x \times 2\pi x \, dx \cdot x$$

$$\frac{\tau_x}{x} = \frac{\tau}{R}$$

$$\tau_x = \frac{\tau}{R} x$$

$$\int dT = \int_r^R \frac{\tau}{R} \cdot x \cdot 2\pi x \, dx \cdot x$$

$$= \int_r^R \frac{2\pi\tau}{R} \cdot x^3 \, dx$$

$$= \frac{2\pi\tau}{R} \int_r^R x^3 dx$$

$$T = \frac{2\pi\tau}{R} \left| \frac{x^4}{4} \right|_r^R$$

$$T = \frac{\pi\tau}{2R} \left( R^4 - r^4 \right)$$

$$T = \frac{\pi\tau}{\cancel{2} \cdot \frac{D}{\cancel{2}}} \left( \frac{D^4}{16} - \frac{d^4}{16} \right)$$

$$T = \frac{\pi\tau}{16} \left( \frac{D^4 - d^4}{D} \right)$$

$$I_p = \frac{\pi}{32} \left( D^4 - d^4 \right) = \frac{\pi}{2} \left( R^4 - r^4 \right)$$

$$T = \frac{\tau}{R} \cdot I_p$$

$$\boxed{\frac{T}{I_p} = \frac{\tau}{R} = \frac{G\theta}{l}}$$

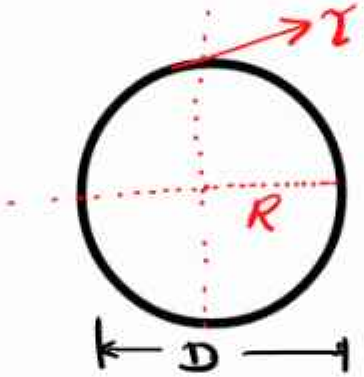
## TORSIONAL RIGIDITY

$$\frac{T}{I_p} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{GI_p} = \quad \theta = \frac{T}{(GI_p/l) \approx k}$$

$$\boxed{k = \frac{GI_p}{l} = \frac{T}{\theta}} \qquad \boxed{k = \frac{T}{\theta}}$$

# A solid shaft of 150mm diameter is used to transmit torque. Find the maximum torque transmitted by the shaft if the maximum shear stress induced to the shaft is 45 N/mm².




Sol$^n$:

$$T = \frac{\pi \tau D^3}{16}$$

$$= \frac{\pi \times 45 \times 150^3}{16}$$

$$\boxed{T = 29820586.52 \text{ N-mm}}$$

# POWER TRANSMITTED BY SHAFTS

Once the expression for torque (T) for a solid or hollow shaft is obtained, power transmitted by the shafts can be determined.

$N$ = r.p.m. of the shaft

$T$ = Mean torque transmitted in N-m

$\omega$ = Aangular speed of shaft

$$\text{Power} = \frac{2\pi N T}{60} \text{ watts}$$

$$\boxed{\text{Power} = \omega T = T \times \omega}$$

\# A solid steel shafts transmits 100 KW at 150 rpm. Determine the suitable diameter of the shaft if the maximum torque transmitted exceeds the mean by 20% in each revolution. The shear stress not to exceed 60MPa. Also find the maximum angle of twist in a length of 4m of the shaft. $G = 80\,GPa$

Sol$^n$. Given data are.

$P = 100$ KW

$l = 4$ m

$N = 150$ rpm

$\tau = 60 MPa$

$G = 80 GPa$

(i) Find dia. of the shaft, if torque is increased by 20%.

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 100 \times 1000}{2\pi \times 150}$$

$$= 6366 \text{ N-m}$$

$$T_{max} \text{ after 20\% increase} = 6366 \times 1.2$$

$$= 7639 \text{ N-m}$$

$$= 7.639 \times 10^6 \text{ N-mm}$$

<u>Shaft diameter</u>

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 7.639 \times 10^6}{\pi d^3} = 60$$

$$d^3 = \frac{16 \times 7.639 \times 10^6}{60\pi}$$

$$d^3 = 648418.4588 \Rightarrow d = 86.6 \text{ mm}$$

<u>Maximum angle of twist</u>

$$\frac{\tau}{R} = \frac{G\theta}{L} \Rightarrow \frac{60}{43.3} = \frac{80 \times 10^3 \theta}{4 \times 1000}$$

$$\theta = \frac{60 \times 4 \times 1000}{43.3 \times 80 \times 10^3}$$

$$\theta = 0.06928 \text{ rad} = 3.97^\circ$$

# In a hollow circular shaft of outer and inner diameters of 20cm and 10cm respectively, the shear stresses is not exceeded 40N/mm2. find the maximum torque which the shaft can safely transmit.

Soln.

$$T = \frac{\pi}{16}\tau\left[\frac{D^4 - d^4}{D}\right]$$

# Find the maximum shear stress indued in a solid circular shaft of diameter 15cm when the shaft transmits 150 KW power at 180 rpm.

Soln.

Apply. $P = \frac{2\pi N T}{60}$

& $T = \frac{\pi \tau D^3}{16}$

# POLAR MODULUS

Polar modulus is the ratio of polar moment of inertia to the radius of shaft.

$$Z_p = \frac{I_p}{R}$$

For solid circular shaft

$$\boxed{I_p = \frac{\pi D^4}{32}}$$

$$Z_p = \frac{\pi D^4}{32R} = \frac{\pi D^4}{32 \times \frac{D}{2}}$$

$$\boxed{Z_p = \frac{\pi D^3}{16}}$$

For hollow shaft

$$\boxed{I_p = \frac{\pi}{32}(D^4 - d^4)}$$

$$Z_p = \frac{\frac{\pi}{32}(D^4 - d^4)}{\frac{D}{2}}$$

$$\boxed{Z_p = \frac{\pi}{16D}(D^4 - d^4)}$$

\# Determine the diameter of solid steel shaft which will transmit 90 kW power at 160 r.p.m. Also determine the length of the shaft if the twist must not exceed $1^\circ$ over the entire length. The maximum shear stress is limited to $60 N/mm^2$. Take the value of modulus rigidity $= 8 \times 10^4 N/mm^2$.

Sol$^n$.

Power $= P = 90$ kW

$N = 160$ rpm

$\tau_{max} = 60$ N/mm$^2$

$G = 8 \times 10^4$ N/mm$^2$

let D is dia of shaft and L is length of shaft.

⇒ Find the dia of shaft.

$$P = \frac{2\pi N T}{60}$$

$$90 \times 10^3 = \frac{2\pi \times 160 \times T}{60}$$

$$T = \frac{90 \times 10^3 \times 60}{2\pi \times 160} = 5371.48 \text{ N-m}$$

$$= 5371.48 \times 10^3 \text{ N-mm}$$

Using torque formula.

$$T = \frac{\pi}{16} \tau D^3$$

$$5371.48 \times 10^3 = \frac{\pi}{16} D^3 \times 60$$

$$D = \left( \frac{16 \times 5371.48 \times 10^3}{\pi \times 60} \right)^{1/3} = 76.96 \text{ mm}$$

⇒ length of the shaft

$$\frac{\tau}{R} = \frac{G\theta}{L} \Rightarrow L = \frac{G\theta R}{\tau}$$

$$= \frac{8\times10^4 \times \frac{\pi}{180} \times \frac{76.96}{2}}{60}$$

$$= \frac{8\times10^4 \times \pi \times 76.96}{180\times2\times60}$$

$$L = 895.5 \text{ mm}$$

# A solid circular shaft transmits 75KW power at 200 rpm. Calculate the shaft diameter, if the twist in the shaft is not to exceed 1° in 2m length of shaft and shear stress is limited to 50N/mm². Take, $G = 1\times10^5$ N/mm².

Solⁿ.

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{\pi}{16}\tau D^3$$

$$\frac{T}{I_p} = \frac{G\theta}{L}$$

# STRENGTH OF SHAFT OF VARYING SECTIONS

When a shaft is made up of different length and of different diameter, the torque transmitted by individual section should be calculated. The strength of such a shaft is the maximum value of these torques.

# A shaft ABC of 500mm length and 40mm external diameter is drilled, for a part of its length AB, to a 20mm diameter and for the remaining length BC to a 30mm dia. drill. If the shear stress is not to exceed 80 N/mm², find the maximum power, the shaft can transmit at a speed of 200 rpm.

Soln.

Total length of shaft = 500 mm

$D = 40mm$

$L_1$ = length of the shaft AB

$L_2$ = length of the shaft BC

$d_1$ = Internal diameter of shaft AB = 20mm

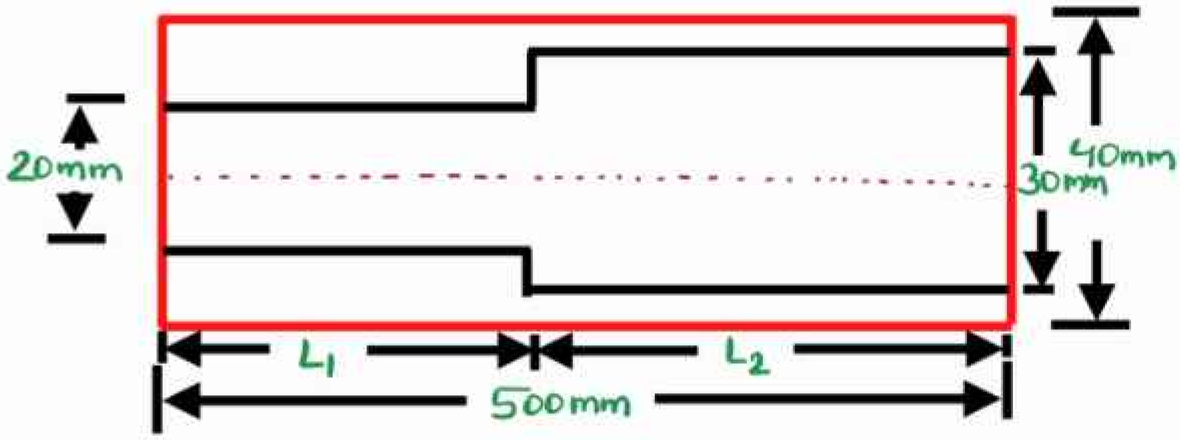
$d_2$ = Internal diameter of shaft BC = 30 mm

$\tau$ = Maximum shear stress = 80 N/mm²

$N$ = Speed = 200 rpm

$T_1$ = Torque transmitted by the shaft AB

$T_2$ = Torque transmitted by the shaft BC




The torque transmitted by hollow shaft given by Equation

$$T = \frac{\pi}{16}\tau\left(\frac{D^4 - d^4}{D}\right)$$

Now, torque transmitted by hollow shaft AB is

$$T_1 = \frac{\pi}{16}\tau\left(\frac{D^4 - d_1^4}{D}\right)$$

$$= \frac{\pi}{16} \times 80\left(\frac{40^4 - 20^4}{40}\right) = 942500\text{ N-mm}$$

$$= 942.500\text{ N-m}$$

torque transmitted by hollow shaft BC

$$T_2 = \frac{\pi}{16}\tau\left(\frac{D^4 - d_2^4}{D}\right)$$

$$= \frac{\pi}{16} \times 80 \times \left( \frac{40^4 - 30^4}{40} \right)$$

$$= 687200 \text{ N-mm} = 687.200 \text{ N-m}$$

Safe torque is considered as minimum value among $T_1$ & $T_2$

$$\text{Safe torque } (T) = 687.200 \text{ N-m}$$

Now, power transmitted $(P) = \frac{2\pi N T}{60}$

$$= \frac{2\pi \times 200 \times 687.200}{60}$$

$$= 14390 \text{ W}$$

$$P = 14.39 \text{ KW}$$

From torque equation

$$\frac{T}{I_p} = \frac{G\theta}{l}$$

$$\theta = \frac{Tl}{G I_p}$$

Angle of twist in shaft AB $= \frac{T l_1}{G (I_p)_1}$

" " " " BC $= \frac{T l_2}{G (I_p)_2}$

Angle of twist in both the shaft will be equal

So, $$\frac{Tl_1}{G(I_p)_1} = \frac{Tl_2}{G(I_p)_2} \Rightarrow \frac{l_1}{(I_p)_1} = \frac{l_2}{(I_p)_2}$$

Now Calculation of polar moment of Inertia

$$(I_p)_1 = \frac{\pi}{32}(D^4 - d_1^4) = \frac{\pi}{32}(40^4 - 20^4)$$

$$(I_p)_2 = \frac{\pi}{32}(D^4 - d_2^4) = \frac{\pi}{32}(40^4 - 30^4)$$

By considering above equation

$$\frac{l_1}{\cancel{\frac{\pi}{32}}(40^4 - 20^4)} = \frac{l_2}{\cancel{\frac{\pi}{32}}(40^4 - 30^4)}$$

$$\frac{l_1}{l_2} = \frac{(40^4 - 20^4)}{(40^4 - 30^4)} = 1.37$$

$$l_1 = 1.37\, l_2$$

As given that, total length of shaft = 500 mm

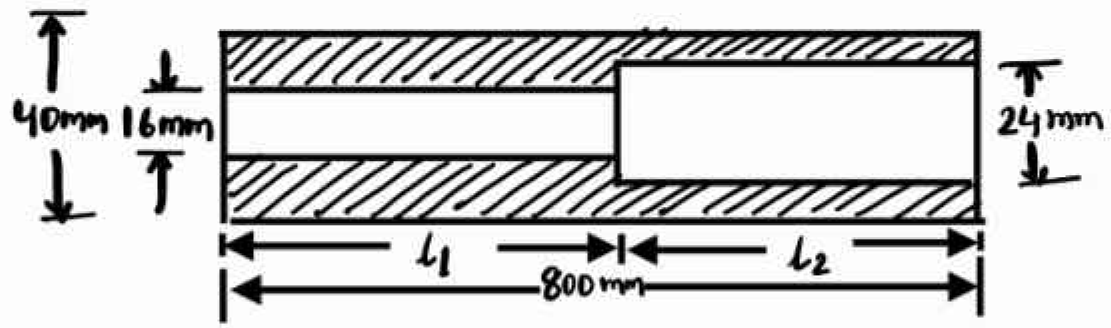
$\therefore \quad l_1 + l_2 = 500$

$$1.37\, l_2 + l_2 = 500 \Rightarrow 2.37\, l_2 = 500$$

$$l_2 = \frac{500}{2.37} = 210.97 \simeq 211 \text{ mm}$$

$$l_1 = 500 - 211 = 289 \text{ mm}$$

# Figure shows a hollow shaft. Determine the maximum power transmitted by the shaft at 200 rpm, if the shear stress in the shaft is not to exceed 70 MPa. Also find the lengths of two portions if the twist produced in the two portions of the shaft are equal.




## SHAFTS IN SERIES AND PARALLEL

Two shafts may be joined in series (or) in parallel.

### Shafts in series

If two shafts is joined in series and single torque is applied, then both shafts will be subjected to same torque.

$$T = (I_P)_1 \cdot \frac{\tau_1}{R_1} = (I_P)_2 \frac{\tau_2}{R_2}$$

Also, $$T = \frac{G_1 (I_P)_1 \theta_1}{l_1} = \frac{G_2 (I_P)_2 \theta_2}{l_2}$$

and Angle of twist $(\theta)$ = angle of twist of shaft 1 + angle of twist of shaft 2

$$\theta = \theta_1 + \theta_2 = \frac{T l_1}{G_1 (I_p)_1} + \frac{T l_2}{G_2 (I_p)_2}$$

## Shaft in parallel

When two shafts are joined in parallel, torque applied to the composite shaft is the sum of torques on the two shafts.

$$T = T_1 + T_2 = \frac{G_1 (I_p)_1 \theta_1}{l_1} + \frac{G_2 (I_p)_2 \theta_2}{l_2}$$

If angle of twist & length of shaft is same

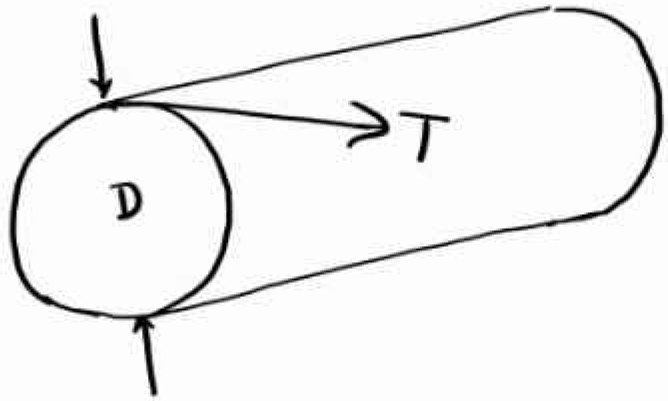
$$T = \frac{\theta}{l}\left[G_1 (I_p)_1 + G_2 (I_p)_2\right]$$

$$\text{Angular twist, } \theta = \frac{T l}{G_1 (I_p)_1 + G_2 (I_p)_2}$$

## COMBINED BENDING AND TORSION

When a shaft is transmitting torque (or) power, it is subjected to shear stresses. At the same time the shaft is also subjected to bending moment due to gravity.

Due to bending moment, bending stresses, bending stresses are also setup in the shaft.




Consider any point on the cross-section of a shaft.

let $T$ = Torque at the section

$D$ = Diameter of the shaft

$M$ = Bending moment at the Section

The torque $T$ will produce shear stress at the point whereas the B.M. will produce bending stress.

$q$ is shear stress at the point produced by torque $T$

$\sigma$ is bending stress at a point produced by B.M.

The shear stress at any point due to torque (T) is given by

$$\frac{q}{r} = \frac{T}{I_p} \qquad \left\{ \because \frac{q}{r} = \frac{\tau}{R} = \frac{T}{I_p} \right\}$$

$$q = \frac{T}{I_p} \times r$$

Bending stress at any point due to bending moment B.M (M)

$$\frac{M}{I} = \frac{\sigma}{y} \text{ (or) } \sigma = \frac{M \times y}{I}$$

From the concept of principal stress & strain

the angle $\theta$ made by the plane of maximum shear with the normal cross-section.

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

The bending stress and shear stress is maximum at a point on the surface of the shaft

$$r = R = \frac{D}{2} \text{ and } y = \frac{D}{2}$$

$\sigma_b$ = Maximum bending stress i.e. on the shaft surface

$$\sigma_b = \frac{M}{I} \cdot \frac{D}{2} = \frac{M}{\frac{\pi}{64} D^4} \times \frac{D}{2}$$

$$= \frac{32M}{\pi D^3}$$

$\tau$ = Maximum Shear Stress i.e. on the surface of shaft

$$= \frac{T}{I_p} \times R = \frac{T}{\frac{\pi}{32} D^4} \times \frac{D}{2} = \frac{16T}{\pi D^3}$$

$$\tan 2\theta = \frac{2\tau}{\sigma} = \frac{2\tau}{\sigma_b} = \frac{2 \times \frac{16T}{\pi D^3}}{\frac{32M}{\pi D^3}} = \frac{T}{M}$$

Major Principal Stress $= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

$$= \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_s^2}$$

$$= \frac{32M}{2 \times \pi D^3} + \sqrt{\left(\frac{32M}{2 \times \pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$

Minor Principal Stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum Shear Stress $= \frac{\text{Major Principal Stress} - \text{Minor principal Stress}}{2}$

$$= \frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)$$

## For Hollow Shaft

$$\text{Major Principal Stress} = \frac{16D}{\pi[D^4-d^4]}\left(M+\sqrt{M^2+T^2}\right)$$

$$\text{Minor Principal stress} = \frac{16D}{\pi[D^4-d^4]}\left(M-\sqrt{M^2+T^2}\right)$$

$$\text{Maximum shear Stress} = \frac{16D}{\pi[D^4-d^4]}\left(\sqrt{M^2+T^2}\right)$$

# A solid shaft of diameter 80mm is subjected to a twisting moment of 8 MN-mm and a bending moment of 5 MN-mm at a point. Determine:
(i) Principal stresses
(ii) Position of the plane on which they act.

**Soln.:** Given data are:-

Diameter of shaft = D = 80 mm

Twisting moment, T = 8 MN-mm = $8\times10^6$ N-mm

Bending moment, M = 5 MN-mm = $5\times10^6$ N-mm

$$\text{Major Principal stress} = \frac{16}{\pi D^3}\left(M+\sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi\times80^3}\left(5\times10^6+\sqrt{(5\times10^6)^2+(8\times10^6)^2}\right.$$

$$= \frac{16\times10^6}{\pi\times80^3}\left(5+\sqrt{25+64}\right)$$

$$= 143.57\ N/mm^2$$

Now, minor principal stress

$$\text{Minor principal stress} = \frac{16}{\pi D^3}\left(M-\sqrt{M^2+T^2}\right)$$

$$= \frac{16}{\pi\times80^3}\left(5\times10^6-\sqrt{(5\times10^3)^2+(8\times10^3)^2}\right)$$

$$= \frac{16\times10^6}{\pi\times80^3}\left(5-\sqrt{25+64}\right)$$

$$= 44.1\ N/mm^2$$

Now position of plane on which major principal stress and minor principal stress act

$$\therefore \quad \tan2\theta = \frac{T}{M} = \frac{8\times10^6}{5\times10^6} = 1.6$$

$$2\theta = \tan^{-1}(1.6)$$

$$2\theta = 58^\circ, 238^\circ$$

$$\theta = 29^\circ, 119^\circ$$

# A maximum allowable shear stress in a hollow shaft of external diameter equal to twice the internal diameter, is $80 N/mm^2$. Determine the diameter of the shaft if it is subjected to a torque of $4\times10^6$ N-mm and a bending moment of $3\times10^6$ N-mm.

Sol$^n$.

Maximum shear stress $= 80 N/mm^2$

Torque, $T = 4\times10^6$ N-mm

Bending moment, $M = 3\times10^6$ N-mm

D = External diameter of shaft

d = Internal diameter of shaft

$d = 2D$

$$\text{Maximum shear stress} = \frac{16D}{\pi(D^4-d^4)}\sqrt{M^2+T^2}$$

$$80 = \frac{16D}{\pi\left[D^4-\left(\frac{D}{2}\right)^4\right]}\sqrt{(3\times10^6)^2+(4\times10^6)^2}$$

$$80 = \frac{16D\times10^6\left[\sqrt{9+16}\right]}{\pi D^4\left[1-\frac{1}{16}\right]} = \frac{16\times10^6\times5}{\pi D^3\times\frac{15}{16}}$$

$$D^3 = \frac{16\times10^6\times5\times16}{\pi\times15\times80} = 0.3395\times10^6$$

$$D = (0.3395 \times 10^6)^{1/3}$$

$$D = 69.78 \text{ mm}$$

$$d = \frac{D}{2} = \frac{69.78}{2} = 34.89 \text{ mm}$$

# The maximum normal stress and the maximum shear stress analysed for a shaft of 150mm diameter under combined bending and torsion, where found to be 120MN/m² and 80 MN/m² respectively. Find the bending moment and torque to which the shaft is subjected.

If the maximum shear stress be limited to 100MN/m², find by how much the torque can be increased if the bending moment is kept constant.

Solution:

$$\sigma_{max} = 120 \text{ MN/m}^2$$

$$(\tau_{max})_1 = 80 \text{ MN/m}^2$$

$$d = 150 \text{mm}$$

$$(\tau_{max})_2 = 100 \text{ MN/m}^2$$

1st Solution

Expression for combined bending and torsion

$$\sigma_{max} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2+T^2}\right]$$

$$\tau_{max} = \frac{16}{\pi d^3}\left[\sqrt{M^2+T^2}\right]$$

$$\sigma_{max} = 120 = \frac{16}{\pi \times (0.15)^3}\left[M + \sqrt{M^2+T^2}\right]$$

$$\tau_{max} = 80 = \frac{16}{\pi \times (0.15)^3}\left[\sqrt{M^2+T^2}\right]$$

$$\sqrt{M^2+T^2} = \frac{\cancel{80}^{5} \times \pi \times 0.15 \times 0.15}{\cancel{16}}$$

$$= 5 \times 0.15 \times 0.15 \times 0.15 \times \pi$$

$$= 0.053$$

$$120 = \frac{16}{\pi \times 0.15 \times 0.15 \times 0.15} \times \left[M + 0.053\right]$$

$$M = \frac{120 \times \pi \times 0.15 \times 0.15 \times 0.15}{16} - 0.053$$

$$\boxed{M = 0.0265 \text{ MNm}}$$

Substituting M in maximum shear stress eqn.

$$\sqrt{M^2+T^2} = 0.053$$

$$M^2+T^2 = 0.053^2$$

$$0.0265^2+T^2 = 0.002809$$

$$T^2 = 0.002809-0.0265^2$$

$$\boxed{T = 0.0459 \text{ MNm}}$$

# An 800mm long shaft with a diameter of 80mm carries a flywheel weighting 4kN at its mid way. The shaft transmits 24kW at a speed of 240rpm. Determine the principal stresses and the maximum shear stress at the end of vertical and horizontal diameter in a plane near the flywheel.

<u>Sol^n.</u>

$L = 800\text{mm}$ $\quad d = 80\text{mm}$

$W = 4\text{kN}$ $\quad P = 24\text{kW}$

$N = 240\text{rpm}$

find principal stresses and maximum shear stresses on vertical and horizontal diameter

Maximum bending moment, $M = \frac{WL}{4}$

$$M = \frac{4000 \times 800}{4}$$

$$= 800 \times 10^3 \text{ Nmm}$$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60P}{2\pi N} = \frac{60 \times 2400}{2\pi \times 240} = 955\,Nm$$

$$T = 955 \times 10^3\,N\,mm$$

At the end of vertical diameter

$$\text{Principal stresses} = \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2+T^2}\right]$$

$$= \frac{16}{\pi \times 80^3}\left[800 \times 10^3 \pm \sqrt{(800 \times 10^3)^2 + (955 \times 10^3)^2}\right]$$

$$\text{Major principal stress} = \frac{16}{\pi \times 80^3}\left[800 \times 10^3 + 10^3\sqrt{800^2+955^2}\right]$$

$$= 20.35\,N/mm^2$$

$$\text{Minor principal stress} = \frac{16}{\pi \times 80^3}\left[800 \times 10^3 - 10^3\sqrt{800^2+955^2}\right]$$

$$= -4.43\,N/mm^2$$

* On the tension side of the shaft (lower end), the principal stresses are 20.35 MPa tension and 4.43 MPa compression

* On the compression side of the shaft (upper end), the principal stresses are 20.35 MPa compression and 4.43 MPa tension.

$$\text{Maximum shear stress} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

$$= \frac{16}{\pi \times 80^3}\sqrt{800^2+955^2} \times 10^3$$

$$= 9.947 \times 10^{-3} \times 1245.8$$

$$= 12.39 \text{ N/mm}^2$$

At the end of horizontal diameter

The bending stress is zero.

$$\text{Torsional shear stress} = \frac{16T}{\pi d^3} = 9.947 \times 10^{-3} \times 955$$

$$= 9.5 \text{ N/mm}^2.$$